Computationally Efficient FBP-Type Image Reconstruction/Segmentation from Cone-Beam Data

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Motivation

Explosive material is serious threat to aviation security, it is important to protect property and people from terrorist attacks

Fig.1 Pan Am flight destroyed by bomb in 1988

Properties of explosive material

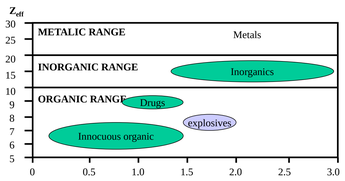
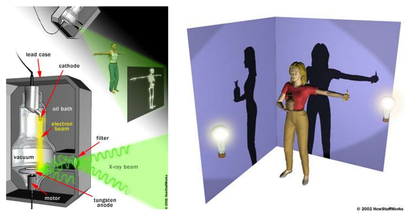


Fig.2 The imaging parameters are density and effective atomic number, explosive material contains high nitrogen, oxygen and moderate carbon

X-ray CT imaging

Fig.3. X-ray has good penetration . Different material absorbs X-ray differently, so the attenuation map can be imaged.



Objectives of explosive detection

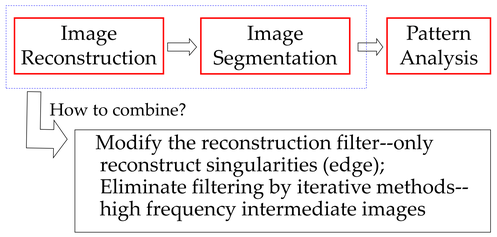
1.Design and implement of novel explosive detection using multi-sensor systems and unconventional approaches involving alternative signatures

2.High probability of detection in each category of explosives

3. Overall low probability of false alarm (< 5%)

4.Enhanced automation for high throughput rate (> 650 bags/hour)

Technique approachs



Idea: eliminate or modify computationally expensive filtering step in the filtered back-projection (FBP) algorithm for simultaneous segmentation and image reconstruction

• Approach 1: Eliminate filtering → *Iterative reprojection-backprojection* with appropriate weighting to guarantee convergence and regularization. The intermediate images are equivalent to segmented images at multiple resolutions

• Approach 2: Modify filtering → *Replace "reconstruction filter" with appropriate differential operators* to produce an "unsharp masking" effect

Cone-beam inversion using Fourier integral operator(FIO)

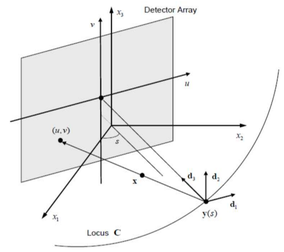


Fig.4. Local coordinate system for cone-beam projection measurement on a planar detector

Method: write the cone-beam transform as a FIO,

f(x) is the object to be reconstruct, A is a standard symbol that is compactly supported in X.

The image is formed by applying the filtered adjoint operator (also FIO) to the projection data.

Exact cone beam reconstruction requires the point spread function to be a Dirac delta function

$$I = \mathcal{F}^{\dagger}\mathcal{Q}[d]$$

Exact recon. filter — *Back-projection* — *Reconstructed image*

Choosing the filter as a differential operator Δ can obtain edge image

$$\hat{I} = \mathcal{F}^{\dagger}\mathcal{A}[d]$$

$$\mathcal{A} = \alpha\mathcal{D} + (1-\alpha)\mathcal{Q}$$

$$\hat{I} = \alpha\tilde{I} + (1-\alpha)I$$

Landweber's iteration scheme

$$f_{k+1} = f_k + \mathcal{D}\mathcal{T}^{\dagger}(g - \mathcal{T}f_k)$$

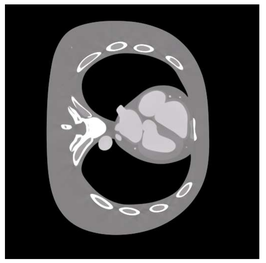
*k*th reconstructed image; Back-projection operator; Forward operator; Measured projection data

D: shaping matrix

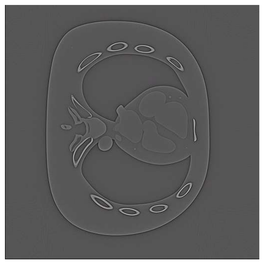
•Accelerate the reconstruction of high frequencies

•Roll off the inverse filter to prevent Gibbs phenomenon

•Preserve the stability of the solution



(a)



(b)

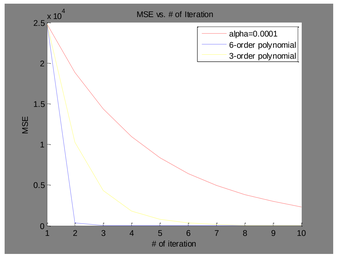


Fig.5. Mean square error versus number of iteration Performance comparison of generalized Landweber's iterative reconstruction using different shaping matrix Red: step length is 0.0001; blue: shaping matrix is a 6-order polynomial of system matrix; yellow: shaping matrix is a 3-order polynomial of system matrix.

Fig.6. Cone-beam reconstruction using differential filter. (a) FDK's method for reconstruction of nCAT phantom (phantom from GE); (b) Image reconstructed by applying differential filter, edge is enhanced, the method is local and computationally efficient.

References

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