Low Order Dynamics Embedding for High Dimensional Time Series with Application to Robust Video Analytics

Researchers: Octavia Camps, Mario Sznaier, Student Name: Fei Xiong
ECE Dept. Northeastern University, Email addresses: {camps, msznaier} @coe.neu.edu

Robust Systems Lab

Abstract

This research considers the problem of finding low order nonlinear embeddings of dynamic data. Our main result is a semi-definite programming based manifold embedding algorithm that exploits the dynamical information encapsulated in the temporal ordering of the sequence to obtain embeddings with lower complexity than those produced by existing algorithms. This algorithm can be applied on appearance–based multi–frame tracking, activity recognition from video and dynamic texture analysis/synthesis.

Relevance

In many instances, the relevant actionable information is encoded in a very small fraction of the available data.

Nonlinear Manifold Embedding (NME) methods provide a tool to reduce the dimensionality of the data.

Existing NME approaches are based on local spatial relationships between data samples and do not exploit the temporal ordering available in video data.

In contrast, in this research we also incorporate the temporal ordering of the data by modeling it as the output of a Wiener system. This approach allows us to bring in powerful concepts from system theory to take full advantage of the available information hidden in the data and obtain simple, yet robust, dynamical descriptors.

Technical Approach

Background

Wiener Model

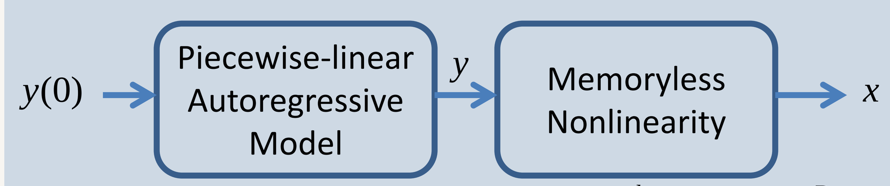


Figure1. Wiener Model. Here, $y \in R^d$ and $x \in R^D$, $d << D$. And $y(0)$ denotes the initial conditions of the autoregressive process.

Regressor Complexity and the Hankel Matrix

Consider a vector process described by an autoregressive model of the form:

$$y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} \cdots + a_n y_{k-n}$$

The order of this model is related to the rank of the Hankel matrix.

$$\mathbf{H}_y = \begin{bmatrix} y_1 & y_2 & \cdots & y_r \\ y_1 & y_3 & \cdots & y_{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_s & y_{s+1} & \cdots & y_{r+s-1} \end{bmatrix}$$

Semidefinite Embedding (SDE)

Define $\mathbf{y} \doteq [y_1 \quad y_2 \quad y_3 \quad \cdots \quad y_N]^T$, $K = \mathbf{y}\mathbf{y}^T$ and neighborhood matrix $\eta \in R^{N \times N}$, $\eta_{ij} \in \{0,1\}$. $\eta_{ij} = 1$ if and only if x_i is regarded as the neighbor of x_j. By assuming that the local isometric property is preserved, Saul [2] proposed the SDE algorithm that

$$\max Trace(K)$$

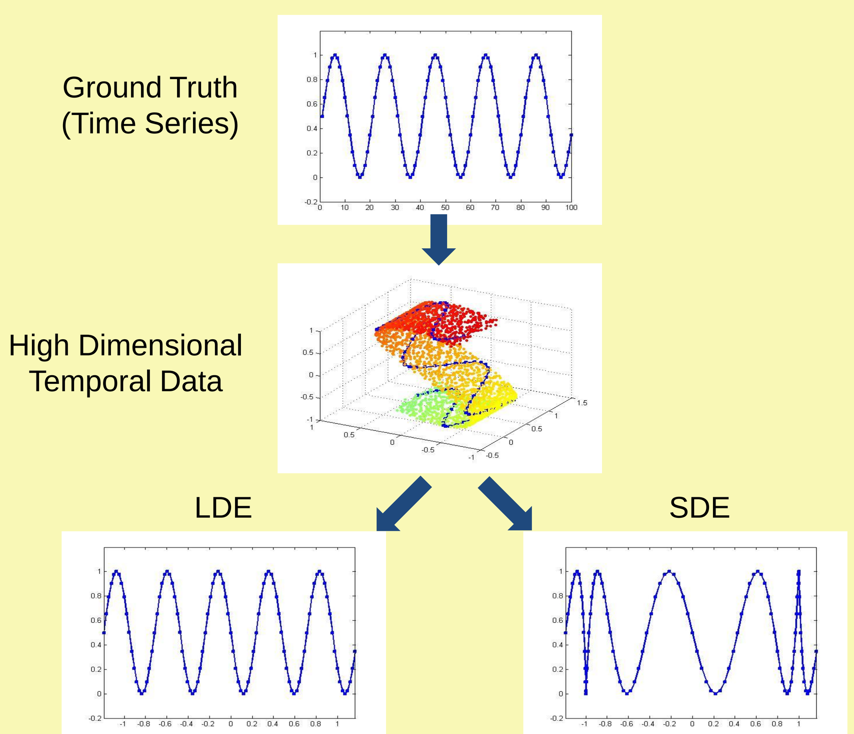
$$\text{s.t.} \quad K_{ii} + K_{jj} + 2K_{ij} = (x_i - x_j)^2 \text{ , if } \eta_{ij} = 1$$

$$K \geq 0$$

$$\sum_{i,j} K_{i,j} = 0$$

Low Order Dynamics Embedding (LDE)

Heuristic



Balancing between maximum Trace(K) and minimum rank(G) unifies the goals of pursuing minimal manifold dimensionality and simpler dynamics of the manifold.

Only pursuing maximum Trace(K) results in the manifold with minimum dimensionality but high dynamic order.

Algorithm

$$\min rank(G) - \lambda Trace(K)$$

$$\text{s.t.} \quad K_{ii} + K_{jj} + 2K_{ij} = (1+\varepsilon)(x_i - x_j)^2, \text{if } \eta_{ij} = 1$$

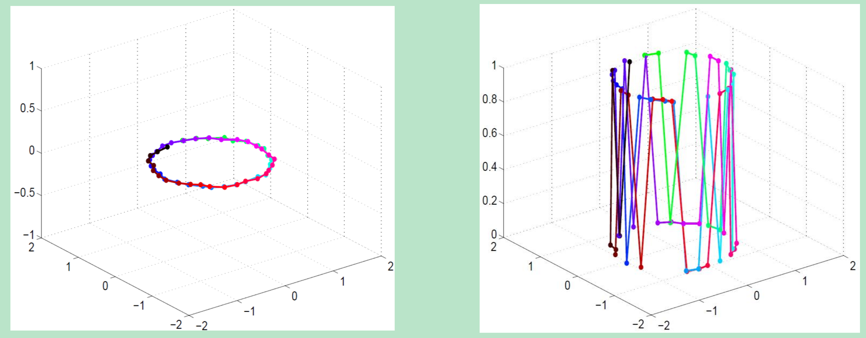
$$K \geq 0 \quad \& \quad \sum_{i,j} K_{ij} = 0 \quad \& \quad G = \sum_{i=1}^{n} K_{i,n-1}$$

Where

$$K_{i,n-1} = \begin{bmatrix} y_i{}^T y_i & y_i{}^T y_{i+1} & \cdots & y_i{}^T y_{i+n-1} \\ y_{i+1}{}^T y_i & y_{i+1}{}^T y_{i+1} & \cdots & y_{i+1}{}^T y_i \\ \vdots & \vdots & \ddots & \vdots \\ y_{i+n-1}{}^T y_i & y_{i+n-1}{}^T y_{i+1} & \cdots & y_{i+n-1}{}^T y_{i+n-1} \end{bmatrix}$$

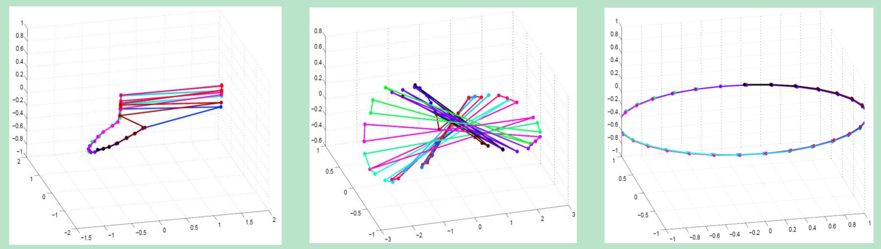
Experiments and Results

Fast Time Varying Data



(a) Original data lying on a 2D circle

(b) 3D data generated by lifting the 2D circle onto two parallel surface

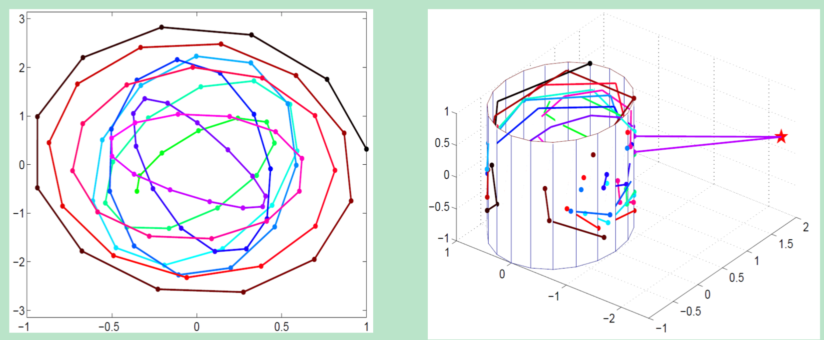


(c) The 2D manifold obtained using DGCM[3].

(d) The manifold obtained using SDE.

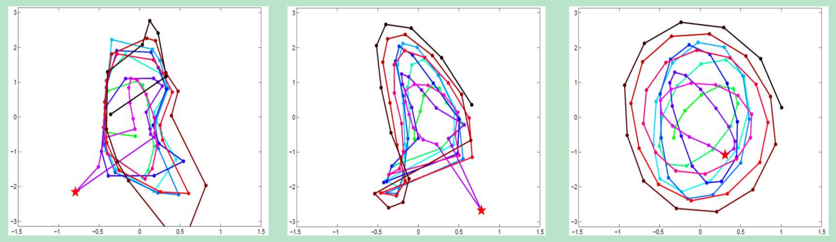
(e) The 2D manifold obtained using LDE.

Data with Outliers



(a) A 2D trajectory synthesized using an 8th order 2output LTI system

(b) 3D data corrupted with an outlier (marked by a star)

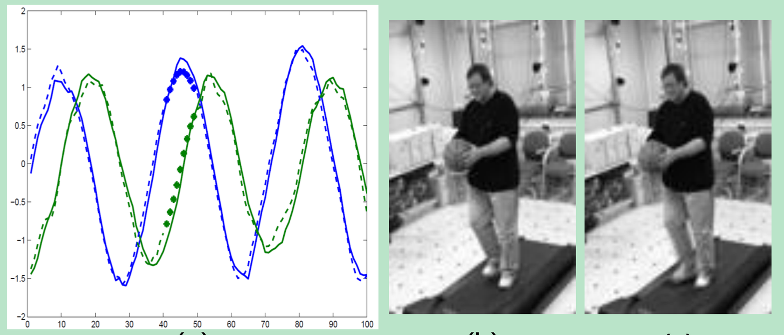


(c) 2D Embedded obtained applying DGCM to the corrupted data

(d) 2D Embedded obtained applying SDE to the corrupted data

(e) 2D Embedded obtained applying LDE to the corrupted data

Sequences with Missing Data

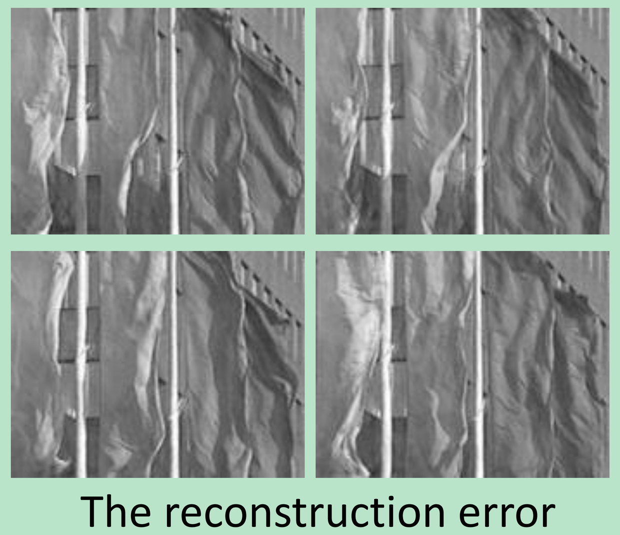


(a) (b) (c)

(a) Manifold interpolation; dashed lines indicate the manifold obtained using 91 frames and rank minimization, the stars indicate interpolated data; solid lines are the ground truth. (b) Sample frame. (c) Reconstructed frame.

Temporal Dynamic Texture

The reconstructed frames



The reconstruction error

Algorithms	Manifold Dimension	Reconstruction Error
DGCM	20	4.8145×10^6
SDE	20	4.3325×10^6
	10	4.8099×10^6
	4	5.0646×10^6
LDE	20	3.3608×10^6
	10	4.0194×10^6
	4	4.3561×10^6

Accomplishments Through Current Year

- LDE reveals the real linear system that is hidden behind the nonlinear distortion.
- A Nonlinear Manifold Embedding approach that is robust to outliers.

Future Work

- Increase accuracy.
- Simplify the algorithm and improve its computational efficiency.

Opportunities for Transition to Customer

Embedding temporal data onto a simple dynamic manifold helps understanding the underlying system and condenses the data into meaningful low dimensional information. This has direct application to video analytics where actions must be taken from terabytes of data.

Publications Acknowledging DHS Support

[1] Fei Xiong, Octavia I. Camps, and Mario Sznaier. Low Order Dynamics Embedding for High Dimensional Time Series. In Proc. 2011 IEEE Int. Conf. Com. Vision (ICCV), Nov. 2011. (to appear)

Other References

[2] K. Weinberger and L. Saul. Unsupervised learning of image manifolds by semidefinite programming. In Computer Vision and Pattern Recognition, 2004. CVPR 2004. Proceedings of the 2004 IEEE Computer Society Conference on, volume 2, pages 988 – 995.
[3] R. Lin, C. Liu, M. Yang, N. Ahuja, and S. Levinson. Learning nonlinear manifolds from time series. In ECCV, volume LNCS 3952, pages 245–256. Springer-Verlag, 2006.
[4] M. Fazel, H. Hindi, and S. Boyd. Log-det heuristic for matrix rank minimization with applications to hankel and euclidean distance matrices. In American Control Conference, 2003. Proceedings of the 2003, volume 3, pages 2156 – 2162.

This material is based upon work supported by the U.S. Department of Homeland Security under Award Number 2008-ST-061-ED0001. The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied of the U.S. Department of Homeland Security.