Extending 2D FDFD Modeling to 2 ½ Dimensions for Realistic Simulation of Millimeter Wave Radar Whole-Body Imaging

Ann W. Morgenthaler and Carey M. Rappaport

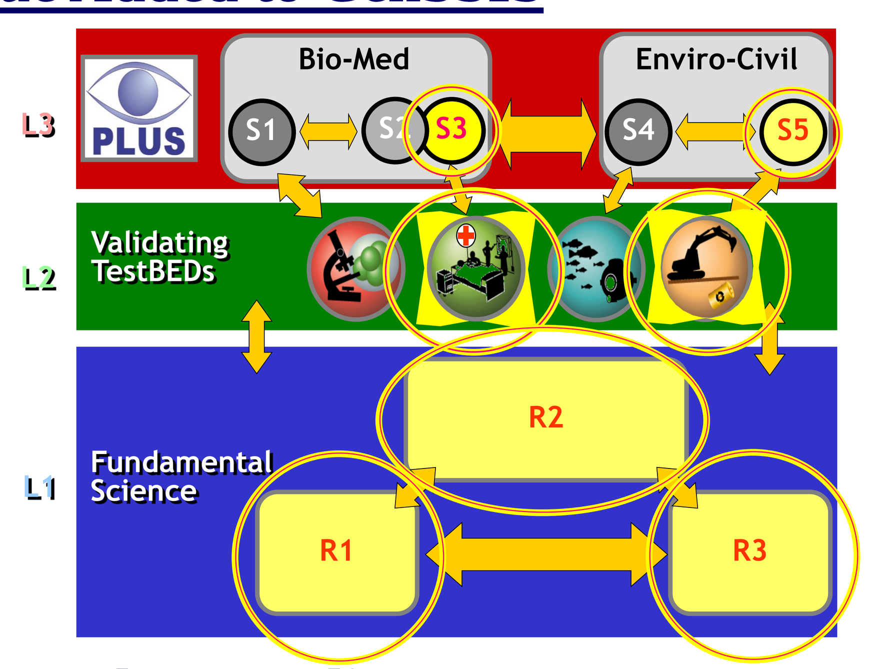
Contact: rappaport@neu.edu

This work is supported by Gordon-CenSSIS, the Gordon Center for Subsurface Sensing and Imaging Systems under the ERC Program of the NSF (Award number EEC-9986821).

Abstract

The 2½ D FDFD (Finite Difference Frequency Domain) algorithm falls between strict 2D and full 3D computational methods and is applicable to geometries which are slowly varying along a preferred axis (chosen to be the *z* axis) and sources which may have a component of wave propagation in that direction. By contrast, 2D FDFD methods require uniform cross-sectional geometries for all values of *z* and "broadside" wave propagation such that $k_z = 0$. We describe the derivation and implementation of the 2 ½ D FDFD algorithm and apply it to the realistic case of a uniform torso cross section illuminated by 2D point sources (lines of current) exterior to the computational grid with arbitrary polarization and propagation directions. The 2 ½ D FDFD simulation requires two wave equations to be solved simultaneously for both longitudinal field components E_z and H_z, rather than solving a single wave equation for E_z (TM) or H_z (TE) as is done in the 2D FDFD algorithm. All four transverse fields may be obtained directly from E_z and H_z, just as H_x and H_y were obtained from E_z in the 2D TM case and E_x and E_y from H_z in the 2D TE case. The resulting FDFD sparse matrix equation $\mathbf{A}\bullet\mathbf{X}=\mathbf{b}$ to be inverted for $\mathbf{X} = [E_z; H_z]$ is four times larger than its 2D analog with 15 nonzero diagonal elements rather than five.

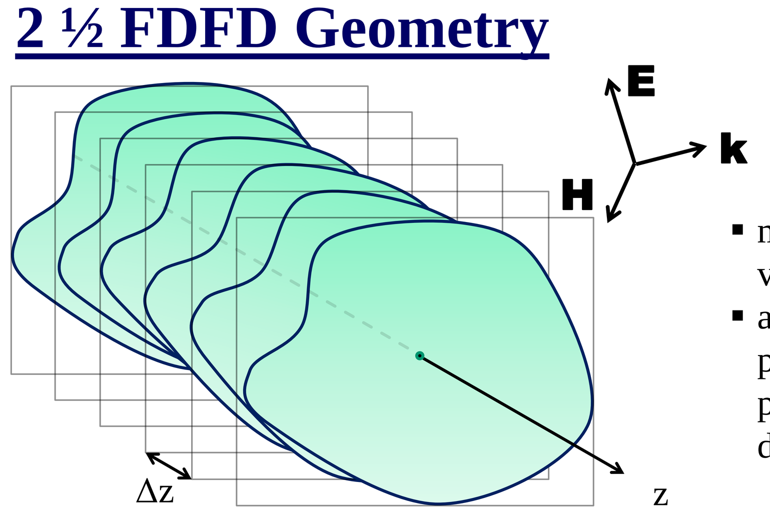
Value Added to CenSSIS



Research to Reality

The 2½ D FDFD algorithm has been developed in order to model the electromagnetic fields arising from realistic geometries and sources in a fast and efficient manner. 2D algorithms are too limiting and full 3D algorithms too slow and/or computationally storage-intensive, so the 2 ½ D algorithm is a practical compromise between speed and model complexity and is applicable to many real world problems (whole-body imaging, tunnel detection etc.) where there is a preferred axis along which the geometry varies slowly. The suite of computational tools being developed at Northeastern University, which includes all three FDFD algorithms, is an important toolkit for any type of electromagnetic scattering simulations, and the FDFD algorithms have particular application as forward models for inverse scattering problems. An interview mode "front end" to the 2D-2½ D FDFD code allows any researcher to simulate problem geometries and sources quickly and then find the resulting fields in a matter of seconds or minutes.

2 ½ FDFD Geometry



- material is slowing varying in *z*
- arbitrary source polarization and propagation direction

Deriving 2 ½ D Equations

- All field components given by: $F_n(x,y,z) = F_{n0}(x,y,az)\, e^{ik_z z}$ where a is small
- Materials (ε & μ) slowly vary in *z*. If they are independent of *z*, then $a = 0$.
- k_z is not necessarily small
- Generalized "TM" wave equation (generalized "TE" equation is dual):

$$\frac{\partial}{\partial x}\left[\left(\frac{k}{k_T}\right)^2\frac{1}{\mu}\frac{\partial E_{z0}}{\partial x} + \frac{2\omega k k_z}{k_T^4}\frac{\partial k}{\partial y}H_{z0} + \left(\frac{k}{k_T}\right)^2\frac{J_{m,y0}}{\mu} - a\left(\frac{k}{k_T}\right)^2\frac{1}{\mu}\frac{\partial E_{x0}}{\partial z}\right] +$$
$$\frac{\partial}{\partial y}\left[\left(\frac{k}{k_T}\right)^2\frac{1}{\mu}\frac{\partial E_{z0}}{\partial y} - \frac{2\omega k k_z}{k_T^4}\frac{\partial k}{\partial x}H_{z0} - \left(\frac{k}{k_T}\right)^2\frac{J_{m,x0}}{\mu} - a\left(\frac{k}{k_T}\right)^2\frac{1}{\mu}\frac{\partial E_{y0}}{\partial z}\right] +$$
$$\frac{k^2}{\mu}E_{z0} + a\frac{i\omega^2 k_z}{k_T^2}\left(\frac{\partial \epsilon}{\partial z}E_{z0} + \epsilon\frac{\partial E_{z0}}{\partial z}\right) = -\frac{i\omega k^2 J_{z0}}{k_T^2} + \frac{i\omega^2 k_z \rho_0}{k_T^2} -$$
$$\frac{2\omega k_z k}{k_T^4}\left(J_{x0}\frac{\partial k}{\partial x} + J_{y0}\frac{\partial k}{\partial y}\right) + a\frac{2\omega k_z k}{k_T^4}\left(\frac{\partial k}{\partial y}\frac{\partial H_{x0}}{\partial z} + \frac{\partial k}{\partial x}\frac{\partial H_{y0}}{\partial z}\right)$$

Always part of the equation | Only present if $k_z \neq 0$ | Only present if $a \neq 0$ | Only present if transverse current sources exist | Only present if $a \neq 0$ and $k_z \neq 0$

- Transverse k_T is given by $k_T = \sqrt{k^2 - k_z^2}$ where $k = \omega\sqrt{\mu\varepsilon}$ is the wave number and k_z is the longitudinal component of *k*.
- J_{x0}, J_{y0}, and J_{z0} are electric current sources
- $J_{m,x0}$, $J_{m,y0}$, and $J_{m,z0}$ are magnetic current sources
- Transverse electric and magnetic fields are given by:

$$E_{x0} = \frac{i}{k_T^2}\left(k_z\frac{\partial E_{z0}}{\partial x} + k\eta\frac{\partial H_{z0}}{\partial y} + k_z J_{m,y0} - k\eta J_{x0} - ak_z\frac{\partial E_{x0}}{\partial z} - ak\eta\frac{\partial H_{y0}}{\partial z}\right)$$
$$E_{y0} = \frac{i}{k_T^2}\left(k_z\frac{\partial E_{z0}}{\partial y} - k\eta\frac{\partial H_{z0}}{\partial x} - k_z J_{m,x0} - k\eta J_{y0} - ak_z\frac{\partial E_{y0}}{\partial z} + ak\eta\frac{\partial H_{x0}}{\partial z}\right)$$
$$H_{x0} = \frac{i}{k_T^2}\left(k_z\frac{\partial H_{z0}}{\partial x} - \frac{k}{\eta}\frac{\partial E_{z0}}{\partial y} + k_z J_{y0} + \frac{k}{\eta}J_{m,x0} - ak_z\frac{\partial H_{x0}}{\partial z} + a\frac{k}{\eta}\frac{\partial E_{y0}}{\partial z}\right)$$
$$H_{y0} = \frac{i}{k_T^2}\left(k_z\frac{\partial H_{z0}}{\partial y} + \frac{k}{\eta}\frac{\partial E_{z0}}{\partial x} - k_z J_{x0} + \frac{k}{\eta}J_{m,y0} - ak_z\frac{\partial H_{y0}}{\partial z} - a\frac{k}{\eta}\frac{\partial E_{x0}}{\partial z}\right)$$

Comparing 2 ½ D Models

Model	Para-meters	General Wave Eqn.	Procedure	Itera-tive?	Coupled E_{z0} and H_{z0}?	Matrix size
Strict 2D - TM	$a = 0$, $k_z = 0$		Solve for E_{z0} using 2D-FDFD solver; find H_{x0} and H_{y0} from transverse equations; only J_{z0} allowed	no	no	**A** matrix is $N_xN_y \times N_xN_y$
Strict 2D - TE	$a = 0$, $k_z = 0$	(Dual)	Solve for H_{z0} using 2D-FDFD solver; find E_{x0} and E_{y0} from transverse equations; only $J_{m,z0}$ allowed	no	no	**A** matrix is $N_xN_y \times N_xN_y$
TM 2 ½ D for broadside sources and slowly varying cross-sectional geometries	$a \neq 0$, $k_z = 0$		Solve for E_{z0} using 2D-FDFD solver; find H_{x0} and H_{y0} from transverse equations; only J_{z0} allowed	yes	no	**A** matrix is $N_xN_y \times N_xN_y$; *z* derivatives will have to be constructed from adjacent cross sections
TE 2 ½ D for broadside sources and slowly varying cross-sectional geometries	$a \neq 0$, $k_z = 0$	(Dual)	Solve for H_{z0} using 2D-FDFD solver; find E_{x0} and E_{y0} from transverse equations; only $J_{m,z0}$ allowed	yes	no	**A** matrix is $N_xN_y \times N_xN_y$; *z* derivatives will have to be constructed from adjacent cross sections
2 ½ D for oblique sources and invariant cross-sectional geometries	$a = 0$, $k_z \neq 0$		Find E_{z0} and H_{z0} simultaneously using 2 ½ D-FDFD solver; find transverse fields; both J_{z0} and $J_{m,z0}$ allowed	no	yes	**A** matrix is $2N_xN_y \times 2N_xN_y$
2 ½ D for oblique sources and slowly-varying cross-sectional geometries	$a \neq 0$, $k_z \neq 0$		Find E_{z0} and H_{z0} simultaneously using 2 ½ D-FDFD solver; find transverse fields; both J_{z0} and $J_{m,z0}$ allowed	yes	yes	**A** matrix is $2N_xN_y \times 2N_xN_y$; *z* derivatives will have to be constructed from adjacent cross sections

Coupled "TM" and "TE" Wave Equation for E_{z0} and H_{z0} ($k_z \neq 0$)

$$\frac{\partial}{\partial x}\left[\alpha^E\frac{\partial E_{z0}}{\partial x} + \beta^E\frac{\partial k}{\partial y}H_{z0}\right] + \frac{\partial}{\partial y}\left[\alpha^E\frac{\partial E_{z0}}{\partial y} - \beta^E\frac{\partial k}{\partial x}H_{z0}\right] + k^2E_{z0} = E_s$$
$$k^2\frac{\partial}{\partial x}\left[\alpha^H\frac{\partial H_{z0}}{\partial x} - \beta^H\frac{\partial k}{\partial y}E_{z0}\right] + k^2\frac{\partial}{\partial y}\left[\alpha^H\frac{\partial H_{z0}}{\partial y} + \beta^H\frac{\partial k}{\partial x}E_{z0}\right] + k^2H_{z0} = H_s$$

where

$$\alpha^E = (k/k_T)^2 \qquad \beta^E = 2\eta k_z k^2/k_T^4$$
$$\alpha^H = (1/k_T)^2 \qquad \beta^H = 2k_z/(\eta k_T^4)$$

First discretization of "TM" wave equation:

$$\frac{\partial}{\partial x}\left[\alpha_{ij}^E\frac{E_{i+1/2,j} - E_{i-1/2,j}}{\Delta x_i} + \beta_{ij}^E\frac{k_{i,j+1/2} - k_{i,j-1/2}}{\Delta y_j}H_{ij}\right] + \frac{\partial}{\partial y}\left[\alpha_{ij}^E\frac{E_{i,j+1/2} - E_{i,j-1/2}}{\Delta y_j} - \beta_{ij}^E\frac{k_{i+1/2,j} - k_{i-1/2,j}}{\Delta x_i}H_{ji}\right] + k_{ij}^2E_{ij} = E_{s,ij}$$

where E_{ij} represents the field E_{z0} at point (i, j) where i and j index the x and y coordinates, respectively, and

$$\Delta x_{i+1/2} = (x_{i+1} - x_i) \qquad \Delta y_{j+1/2} = (y_{j+1} - y_j)$$
$$\Delta x_i = (x_{i+1/2} - x_{i-1/2}) \qquad \Delta y_j = (y_{j+1/2} - y_{j-1/2})$$

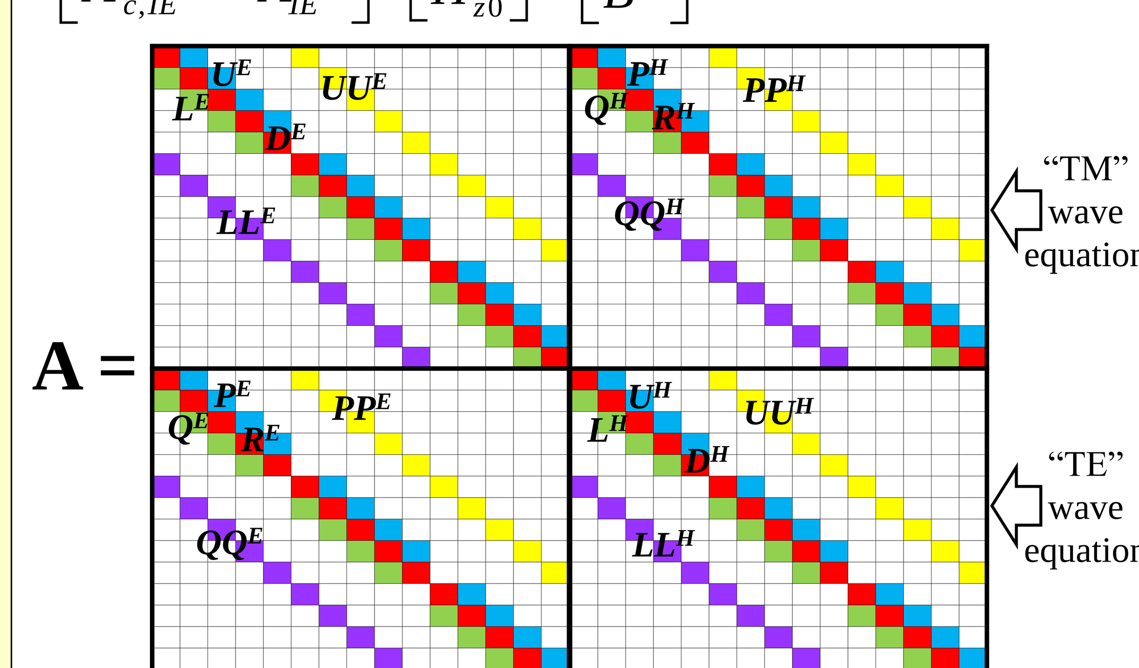
Second discretization of "TM" wave equation:

$$L_{ij}^E E_{i,j-1} + U_{ij}^E E_{i,j+1} + D_{ij}^E E_{ij} + LL_{ij}^E E_{i-1,j} + UU_{ij}^E E_{i+1,j} + Q_{ij}^H H_{i,j-1} + P_{ij}^H H_{i,j+1} + R_{ij}^H H_{ij} + QQ_{ij}^H H_{i-1,j} + PP_{ij}^H H_{i+1,j} = B_{ij}^E$$

$$L_{ij}^E = \frac{\alpha_{i,j-1/2}^E}{\Delta y_{j-1/2}\Delta y_j} \qquad Q_{ij}^H = \frac{\beta_{i,j-1/2}^E\left(k_{i+1/2,j-1/2} - k_{i-1/2,j-1/2}\right)}{2\Delta x_i\Delta y_j}$$
$$U_{ij}^E = \frac{\alpha_{i,j+1/2}^E}{\Delta y_{j+1/2}\Delta y_j} \qquad P_{ij}^H = -\frac{\beta_{i,j+1/2}^E\left(k_{i+1/2,j+1/2} - k_{i-1/2,j+1/2}\right)}{2\Delta x_i\Delta y_j}$$
$$LL_{ij}^E = \frac{\alpha_{i-1/2,j}^E}{\Delta x_{i-1/2}\Delta x_i} \qquad QQ_{ij}^H = -\frac{\beta_{i-1/2,j}^E\left(k_{i-1/2,j+1/2} - k_{i-1/2,j-1/2}\right)}{2\Delta x_i\Delta y_j}$$
$$UU_{ij}^E = \frac{\alpha_{i+1/2,j}^E}{\Delta x_{i+1/2}\Delta x_i} \qquad PP_{ij}^H = \frac{\beta_{i+1/2,j}^E\left(k_{i+1/2,j+1/2} - k_{i+1/2,j-1/2}\right)}{2\Delta x_i\Delta y_j}$$
$$D_{ij}^E = -\left(L_{ij}^E + LL_{ij}^E + U_{ij}^E + UU_{ij}^E\right) + h^2k_{ij}^2 \qquad R_{ij}^H = \left(Q_{ij}^H + QQ_{ij}^H + P_{ij}^H + PP_{ij}^H\right)$$

Schematic of Coupled A Matrix

$$\begin{bmatrix} A_{TM} & A'_{c,TM} \\ A'_{c,TE} & A_{TE} \end{bmatrix} \bullet \begin{bmatrix} E_{z0} \\ H_{z0} \end{bmatrix} = \begin{bmatrix} B^E \\ B^H \end{bmatrix}$$ is 2 ½ D FDFD matrix equation



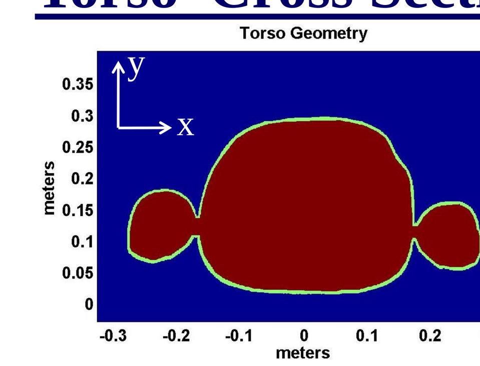
$A_{TM} \bullet E_{z0} = B^E$ is 2D-TM FDFD matrix equation

$A_{TE} \bullet H_{z0} = B^H$ is 2D-TE FDFD matrix equation

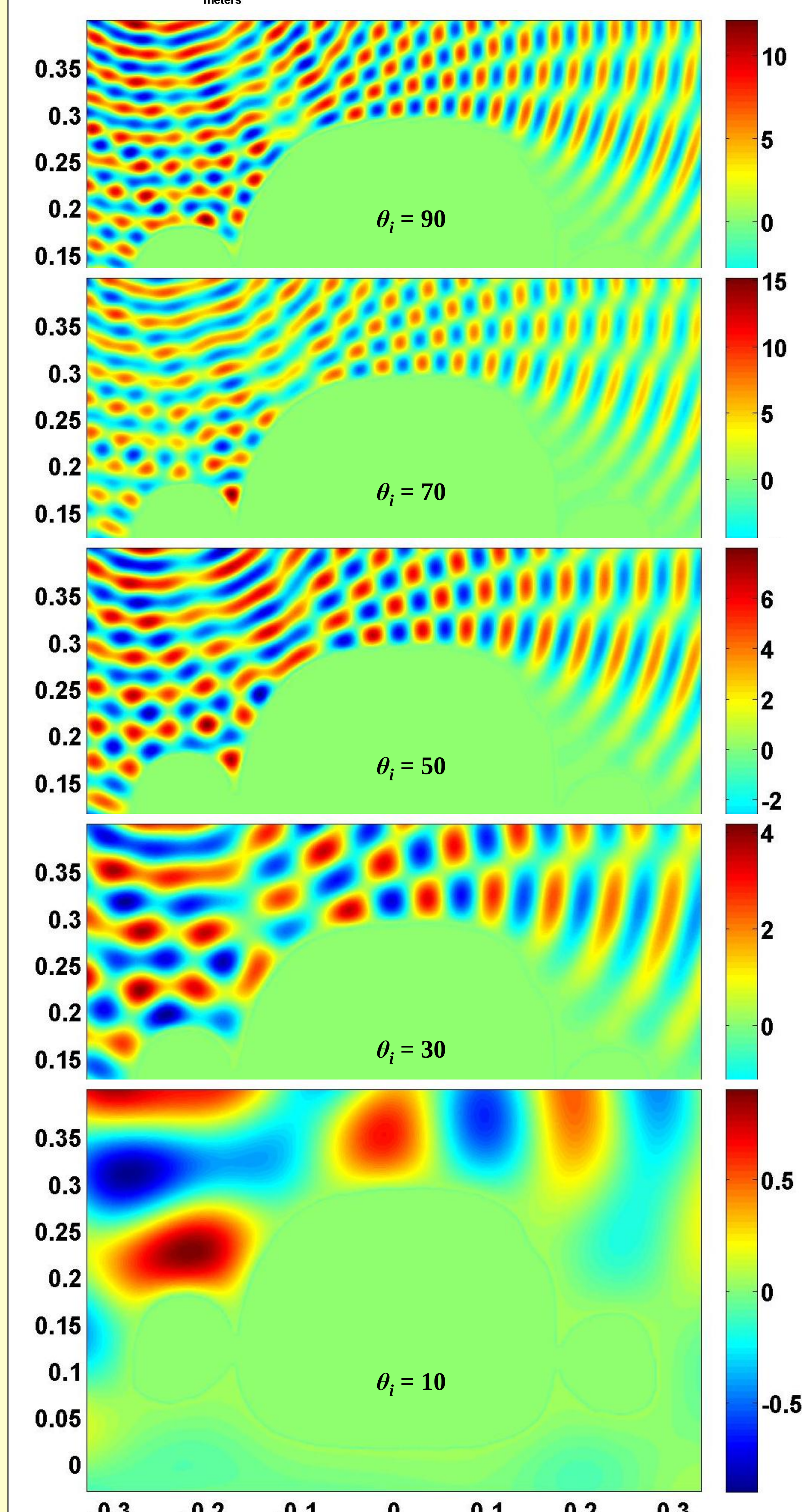
For a point source at $i = i_0$ and $j = j_0$:

$$\mathbf{B} = [B^E; B^H] = [-\eta I^E; I^H/\eta]\, ik(k/k_T)^2 e^{ik_z z}\delta_{i,i_0}\delta_{j,j_0}$$

Point Source Scattering from a Realistic Torso Cross Section for Various k_z values



- f = 10 GHz
- h = 0.513 mm (10 points per λ in skin)
- geometry: blue = air, green = skin, red = metal
- $\varepsilon_{skin} = (30 + 13.5\,i)\,\varepsilon_0$
- current sources: $I^E = 1$ and $I^H = 0$
- ξ = 30 deg (tilt angle from normal in *x*-*y* plane)
- point current source located at (x, y) = (−0.25, 0.5) meters
- $k_z = 2\pi f\sqrt{\mu_0\varepsilon_0}\cos\theta_i$



References

1. Morgenthaler, A., *Coupled Wave Equations for 2 ½ D FDFD Simulations*, 2010.
2. Rappaport, C., Dong, Q., Bishop, E., Morgenthaler, A., and Kilmer, M., "Finite Difference Frequency Domain (FDFD) Modeling of Two Dimensional TE Wave Propagation and Scattering," *URSI Symp. Digest* Pisa, Italy, May 16-18, 2004, pp. 1134-1136.
3. Berenger, J. P., "A perfectly matched layer for the absorption of electromagnetic wave," J. Comp. Phys., vol. 114, pp. 185-200, 1994.